

Problem 2.10

[Difficulty: 2]

2.10 The velocity for a steady, incompressible flow in the xy plane is given by $\vec{V} = \hat{i}A/x + \hat{j}Ay/x^2$, where $A = 2 \text{ m}^2/\text{s}$, and the coordinates are measured in meters. Obtain an equation for the streamline that passes through the point $(x, y) = (1, 3)$. Calculate the time required for a fluid particle to move from $x = 1 \text{ m}$ to $x = 2 \text{ m}$ in this flow field.

Given: Velocity field

Find: Equation for streamline through (1,3)

Solution:

For streamlines

$$\frac{v}{u} = \frac{dy}{dx} = \frac{A \cdot \frac{y}{x^2}}{\frac{A}{x}} = \frac{y}{x}$$

So, separating variables

$$\frac{dy}{y} = \frac{dx}{x}$$

Integrating

$$\ln(y) = \ln(x) + c$$

The solution is

$$y = C \cdot x \quad \text{which is the equation of a straight line.}$$

For the streamline through point (1,3)

$$3 = C \cdot 1 \quad C = 3 \quad \text{and} \quad y = 3 \cdot x$$

For a particle

$$u_p = \frac{dx}{dt} = \frac{A}{x} \quad \text{or} \quad x \cdot dx = A \cdot dt \quad x = \sqrt{2 \cdot A \cdot t + c} \quad t = \frac{x^2}{2 \cdot A} - \frac{c}{2 \cdot A}$$

Hence the time for a particle to go from $x = 1$ to $x = 2 \text{ m}$ is

$$\Delta t = t(x = 2) - t(x = 1) = \frac{(2 \cdot \text{m})^2 - c}{2 \cdot A} - \frac{(1 \cdot \text{m})^2 - c}{2 \cdot A} = \frac{4 \cdot \text{m}^2 - 1 \cdot \text{m}^2}{2 \times 2 \cdot \frac{\text{m}^2}{\text{s}}} \quad \Delta t = 0.75 \cdot \text{s}$$